Student Name: ______________________

Teacher: ______________________

2012
TRIAL HSC
EXAMINATION

Mathematics

Examiners

Mrs P. Biczo, Mr S. Faulds, Ms S. Cupac, Mr S. Gee and Mr J. Dillon

General Instructions

- Reading time - 5 minutes.
- Working time - 3 hours.
- Write using black or blue pen. Diagrams may be drawn in pencil.
- Board-approved calculators and mathematical templates may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11-16.
- Start each question in a separate answer booklet.
- Put your student number on each booklet.

Total marks - 100

Section I

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II

90 marks

- Attempt Questions 11-16. Each of these six questions are worth 15 marks
- Allow about 2 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 Which of the following correctly shows the numeral 0.000 015 in scientific notation:

A 15×10^{-5} **B** 15×10^{6} **C** 1.5×10^{-6} **D** 1.5×10^{-5}

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2 When the denominator is rationalised, $\dfrac{1}{\sqrt{5}-\sqrt{3}} =$

A $\dfrac{\sqrt{5}-\sqrt{3}}{2}$ **B** $\dfrac{\sqrt{5}-\sqrt{3}}{16}$ **C** $\dfrac{\sqrt{5}+\sqrt{3}}{2}$ **D** $\sqrt{5}+\sqrt{3}$

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3 $x^{\frac{1}{2}} =$

A $\dfrac{1}{x^2}$ **B** $\dfrac{x}{2}$ **C** $\dfrac{1}{\sqrt{x}}$ **D** $\sqrt{x}$

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4 The solution to $|x+1| \le 3$ is:

A $-4 \le x \le 2$ **B** $-2 \le x \le 2$ **C** $-2 \le x \le 4$ **D** $x \le -4$ and $x \ge 2$

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5 The solutions of $x^2+7x-3=0$ are

A $x=\dfrac{-7\pm\sqrt{37}}{2}$ **B** $x=\dfrac{7\pm\sqrt{37}}{2}$ **C** $x=\dfrac{-7\pm\sqrt{61}}{2}$ **D** $x=\dfrac{7\pm\sqrt{61}}{2}$

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6 The solution of $2-x<5$ is

A $x<-3$ **B** $x>-3$ **C** $x<3$ **D** $x>3$

---

7 The number 0.07086 rounded to 3 significant figures is:

A 0.070 **B** 0.071 **C** 0.0708 **D** 0.0709

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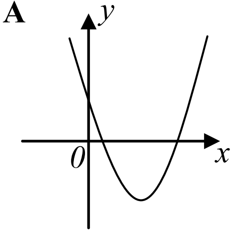
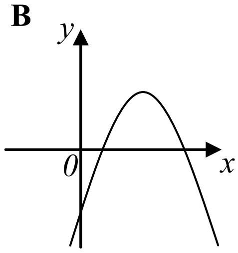
8 Which of the following parabolas could have the equation $y=(x-5)(x-1)$?

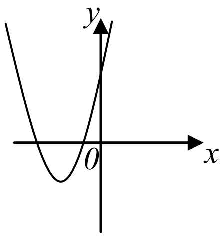
A



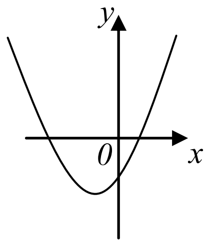
B



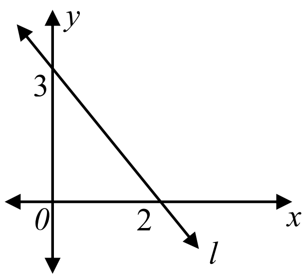
C



D



9



The angle of inclination of the line l with the x axis, to the nearest degree, is

A 34° **B** 56° **C** 124° **D** 146°

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10 The solution for x which satisfies the pair of simultaneous equations: $\begin{cases} x+2y=3 \\ x-y=6 \end{cases}$ is:

A $x=-3$ **B** $x=-1$ **C** $x=5$ **D** $x=9$

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section

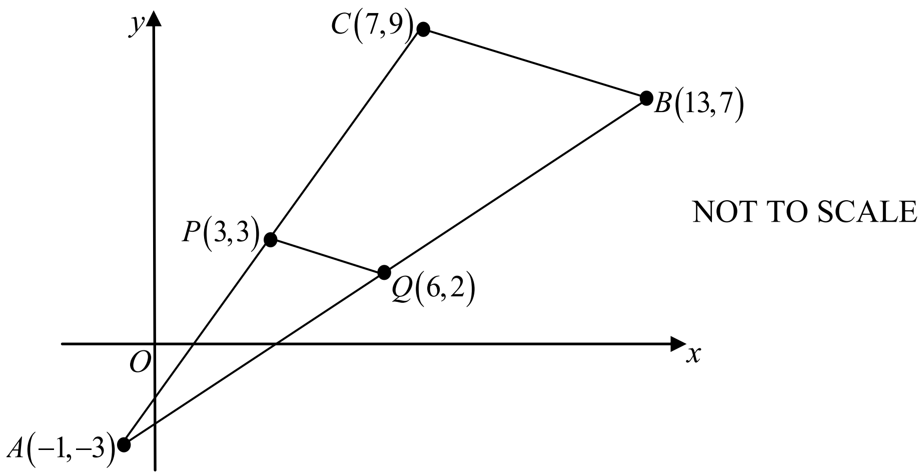
***Answer each question in a new answer booklet*.**

All necessary working should be shown in every question.

Question 11 (15 marks) **Start a new answer booklet** **Marks**

(a) Solve $2x^2 + x - 6 \geq 0$. **2**

(b) In the diagram A, B and C are the points $(-1,-3)$, $(13,7)$ and $(7,9)$ respectively. The points $P(3,3)$ and $Q(6,2)$ are the midpoints of AC and AB respectively.



(i) Find the gradient of PQ. **1**

(ii) Prove that ΔABC is similar to ΔAQP. **3**

(iii) Show that the equation of the line PQ is $x + 3y - 12 = 0$. **1**

(iv) Find the exact length of PQ. **1**

(v) Find the perpendicular distance of the point A to the line PQ. **2**

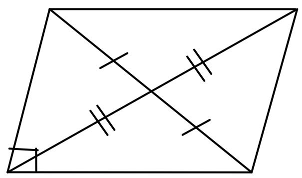
(vi) Hence, find the area of ΔAPQ. **1**

Question 11 continued over the page

(c) On a number plane, shade the region for which the following inequalities hold simultaneously, clearly marking any points of intersection. **2**

$$x^2 + y^2 \leq 4$$
$$x + y \leq 2$$

(d) Give the best name for the quadrilateral shown. Justify your answer, by commenting on the significance of the information given. **2**



Question 12 (15 marks) **Start a new answer booklet** **Marks**

(a) (i) Copy and complete the table of values shown below in your answer booklet for the function $y = x\sin x$. The values in the table should be given in ***exact*** *form*. **1**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{\pi\sqrt{2}}{8}$			0

(ii) Using Simpson's Rule with 5 function values find an approximation to the integral: **2**

$$\int_0^{\pi} x \sin x \, dx$$

(b) The area enclosed by the curve $y = \sqrt{r^2 - x^2}$ is rotated about the x-axis.

(i) What is the name given to the solid that is generated? **1**

(ii) Explain why the volume of the solid of revolution between $x = -r$ and $x = r$ is twice the integral

$$\int_0^r \pi(r^2 - x^2)dx$$ **1**

(iii) Show that the volume of the solid formed is $\frac{4\pi r^3}{3}$. **2**

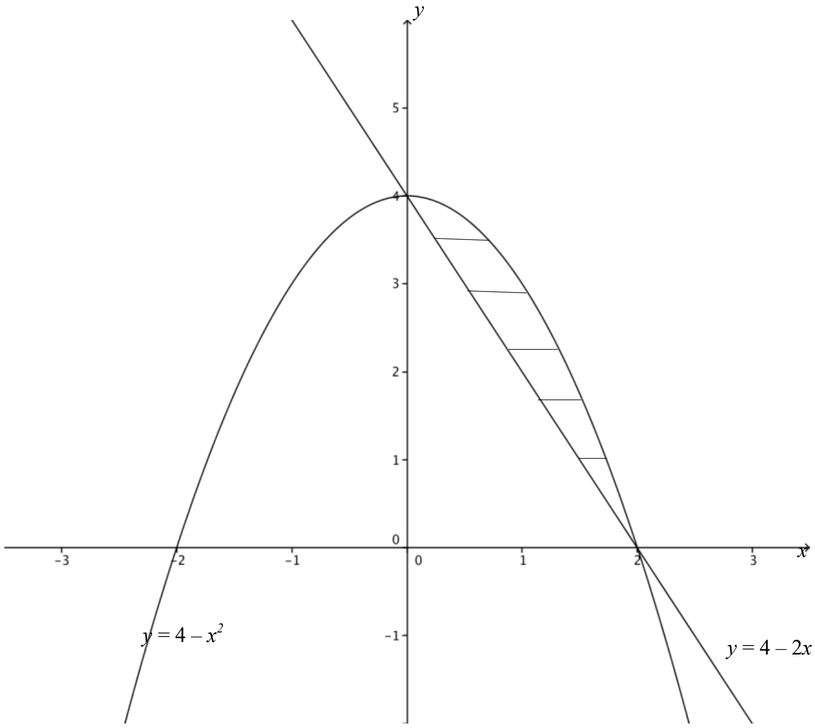
(c) Find the function $y = f(x)$ if $f''(x) = 6x$, $f'(0) = -2$ and $f(1) = 0$. **2**

(d) Given that $\int f'(x)e^{f(x)}dx = e^{f(x)} + c$, find $\int 4x e^{x^2} dx$ **1**

Question 12 continued over the page

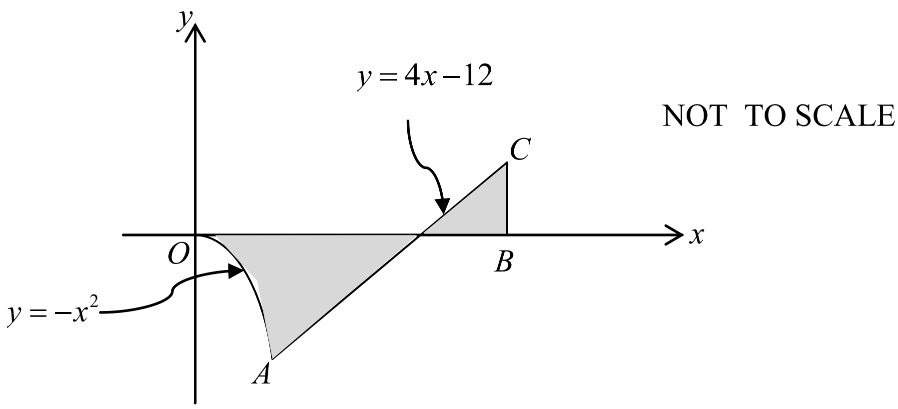
Question 12 continued

(e) The graph below shows the functions $y = 4 - x^2$ and $y = 4 - 2x$. The area enclosed by these functions has been shaded.



The shaded area is revolved around the y-axis. Calculate the volume of the solid generated, leaving your answer in exact form. **3**

(f) The shaded region is bounded by the line $x = 0$, $x = 4$, the curve $y = -x^2$, the line $y = 4x - 12$ and the x axis, as in the diagram. A has co-ordinates $(2, -4)$.



What is the area of the shaded region? **2**

Question 13 (15 marks) **Start a new answer booklet** **Marks**

(a) Let $\log_a 2 = x$ and $\log_a 3 = y$.
Find an expression for $\log_a 12$ in terms of x and y. **2**

(b) Find the equation of the tangent to the curve $y = 3\ln x + 2$ at the point where $x = 1$. **2**

(c) Show that $\dfrac{d}{dx}\left(\dfrac{e^{2x}}{2x+1}\right) = \dfrac{4xe^{2x}}{(2x+1)^2}$. **2**

(d) Solve the following equation for x: $2e^{3x} - e^{2x} = 0$. **2**

(e) (i) Show that $\dfrac{d}{dx}\left((x-1)\log_e 2x\right) = \dfrac{x-1}{x} + \log_e 2x$. **1**

(ii) Hence, or otherwise, show that $\displaystyle\int_{\frac{1}{2}}^{1} \log_e 2x \, dx = \log_e 2 - \frac{1}{2}$. **3**

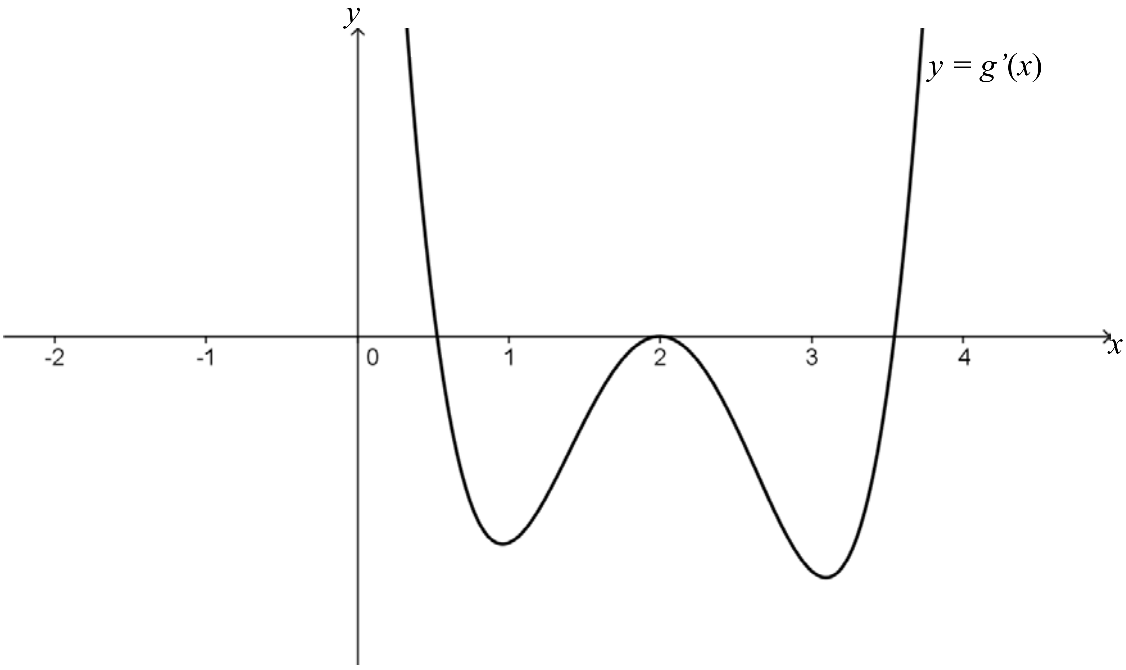
(f) A horizontal line is drawn to cut the graphs $y = e^x$ and $y = \dfrac{1}{2}e^x$ at the points C and D.

(i) Draw a graph to show this information. **1**

(ii) Show that the distance CD is constant (that is, it does not depend on the position where the horizontal line is drawn). **2**

Question 14 (15 marks) **Start a new answer booklet** **Marks**

(a) Shown below is a graph of the derivative function $y = g'(x)$.



(i) If the function $y = g(x)$ were to be drawn using information from the graph above, what feature would exist on the graph at $x = 2$? Justify your answer using your knowledge of differential calculus. **2**

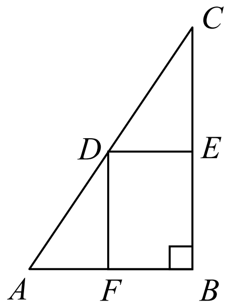
(ii) In your answer booklet, draw a neat sketch of a possible function for $y = g(x)$, given that $g(0) = 0$. **2**

(iii) Explain why it is necessary to give a point on $y = g(x)$ (ie. $g(0) = 0$) in part (ii) in order for the graph to be drawn. **1**

Question 14 continued over the page

Question 14 continued

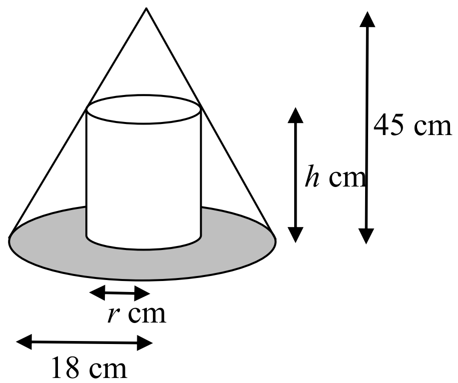
(b)



In the diagram above, ΔABC is right-angled at B. $DEBF$ is a rectangle inscribed in ΔABC.

(i) Briefly explain why $\frac{DF}{CB} = \frac{AF}{AB}$. (Note: It is not necessary to complete a geometric proof to answer this question.) **2**

(ii)



The above diagram shows a right circular cone with perpendicular height, 45cm and radius, 18cm. Inscribed within the cone is a cylinder of height, h cm and radius, r cm.

Explain how the diagram and relationship given in part (i) can be related to the cone and cylinder above, and hence show that: **2**

$$h = \frac{5(18-r)}{2}$$

(iii) Find the value of r that will make the volume of the cylinder inscribed in the given cone a maximum. **3**

(c) (i) Show that the function $f(x) = 3x^2 - 6x + 7$ is positive for all real values of x. **1**

(ii) Hence, or otherwise, show that the function $g(x) = x^3 - 3x^2 + 7x - 10$ is increasing for all values of x. Justify your answer. **2**

Question 15 (15 marks) **Start a new answer booklet** **Marks**

(a) The first three terms of an arithmetic sequence are 7, 11 and 15.

(i) Is 111 a term in this sequence? Justify your answer, by performing appropriate calculations. **2**

(ii) Find the sum of the first twenty-six terms. **2**

(b) After starting work, James decides to invest \$2400 in a superannuation fund at the beginning of each year, commencing on 1 January 2012. The superannuation fund pays an interest rate of 7.25% per annum which compounds annually.

(i) What will be the value of James' superannuation at the end of three years? **2**

(ii) James visited a financial advisor who told him he needs \$500 000 in order to retire comfortably after 40 years service. Will James be able to retire comfortably at his current contribution rate? Justify your answer, by performing appropriate calculations. **2**

(c) Consider the geometric series

$$1 + (\sqrt{11} - 3) + (\sqrt{11} - 3)^2 + \ldots$$

(i) Explain why the geometric series has a limiting sum. **1**

(ii) Find the exact value of the limiting sum. Write your answer with a rational denominator. **2**

(d) Lisa and Monika play a tennis match against each other. The first player to win 2 sets wins the match. The probability that Monika wins any set is 60%.

(i) What is the probability that the game will last two sets only? **2**

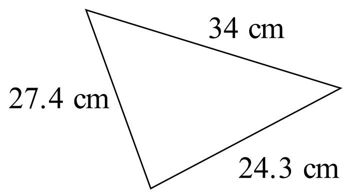
(ii) What is the probability that Lisa wins the match? **2**

Question 16 (15 marks) **Start a new answer booklet** **Marks**

(a) Solve for θ in the given domain:

$$2\sin\theta - \sqrt{3} = 0 \quad \text{for } 0° \le \theta \le 360°$$ **2**

(b) Find the size of the smallest angle in the triangle below to the nearest minute. **2**



(c) A 15 cm arc on the circumference of a circle subtends an angle of $\frac{\pi}{5}$ at the centre of the circle. Find

(i) the radius of the circle, as an exact answer. **1**

(ii) the area of the **major** sector **formed** to one decimal place. **2**

(d) Show that the exact value of $\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{3}\right)$ is $\frac{\sqrt{2}+\sqrt{3}}{2}$ **2**

(e) Show that $\frac{\left(1+\tan^2\theta\right)\cot\theta}{\operatorname{cosec}^2\theta} \equiv \tan\theta$ **2**

(f) For the parabola $y = \frac{x^2}{8} - 1$ **explain why**:

(i) the vertex is $(0, -1)$ **1**

and

(ii) the focal length is 2 units **1**

(g) Find the value(s) of m for which the equation

$$4x^2 - mx + 9 = 0$$

has exactly one real root. **2**

End of examination

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \; n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \; x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \; a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \; x > 0$

Year 12 Mathematics

Section I - Answer Sheet

Student Number ______________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

correct ↙

A ● B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Year 12 Trial HSC Section A	Mathematics Solutions and Marking Guidelines	Examination 2012

Outcomes Addressed in this Question

P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometrical techniques
P5 Understands the concept of a function and the relationship between a function and its graph

Outcome	Solutions	Marking Guidelines
P5	1) $0.000015 = 1.5\times10^{-5}$. $\quad\therefore D.$	1 mark each
P3	2) $\dfrac{1}{\sqrt{5}-\sqrt{3}} = \dfrac{1}{\sqrt{5}-\sqrt{3}}\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \dfrac{\sqrt{5}+\sqrt{3}}{5-3} = \dfrac{\sqrt{5}+\sqrt{3}}{2}$ $\quad\therefore C.$	
P4	3) $x^{\frac{1}{2}} = \sqrt{x}$. $\quad\therefore D.$	
P4	4) $\lvert x+1\rvert \le 3$ $\therefore -3 \le x+1 \le 3$ $\therefore -4 \le x \le 2$ $\quad\therefore A.$	
P4	5) $x^2+7x-3=0$ $x=\dfrac{-7\pm\sqrt{49-4\times1\times-3}}{2}$ $x=\dfrac{-7\pm\sqrt{61}}{2}$ $\quad\therefore C.$	
P4	6) $2-x<5$ $\therefore -x<3$ $\therefore x>-3$ $\quad\therefore B.$	
P4	7) 0.07086 = 0.0709 $\quad\therefore D.$	
P5	8) $y=(x-5)(x-1)$ cuts the x axis at 1 and 5 (when y = 0), and is concave up. $\quad\therefore A.$	
P4	9) $m=\tan\theta$, where θ is the angle of inclination $\therefore \tan\theta = \dfrac{-3}{2}$ As tan negative, θ is in quadrant 2. Basic angle is $56°$, $\therefore \theta = 180-56 = 124°$. $\quad\therefore C.$	

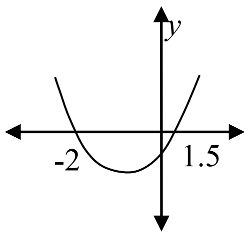
P4	10) Subtracting $\begin{array}{l} x+2y=3 \\ x-y=6 \end{array}$ $-$ $3y=-3$ $\therefore y=-1.$ Substituting in $x+2y=3$, $x-2=3$ $\therefore x=5.$ $\therefore C.$	

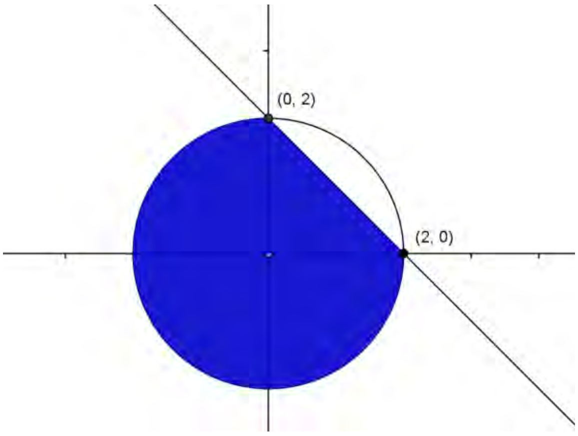
Year 12 Trial Question No.11	Mathematics Solutions and Marking Guidelines	Examination 2012

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, **geometry**, probability, trigonometry and series to solve problems

P4 Chooses and applies appropriate **arithmetic**, algebraic, graphical, trigonometric and geometric techniques

Outcome	Solutions	Marking Guidelines
H5	a) $2x^2+x-6\ge 0.$ $2x^2+4x-3x-6\ge 0$ $2x(x+2)-3(x+2)\ge 0$ $(x+2)(2x-3)\ge 0$ From the graph $x\le -2$ and $x\ge \frac{3}{2}$ -12 | 4, -3, 1 	2 marks : correct solution 1 mark : substantial progress towards correct solution
H5	b) (i) $mPQ=\frac{-1}{3}$	1 mark: correct answer
	(ii) In ΔABC and ΔAQP. $\angle A$ is common $\frac{AP}{AC}=\frac{1}{2}$ (Given P is the midpoint of AC) $\frac{AQ}{AB}=\frac{1}{2}$ (Given Q is the midpoint of AB) $\therefore$ ΔAQP is similar to ΔABC (a pair of sides in the same ratio and included angles equal)	3 marks: correct solution 2 marks: substantially correct solution 1 mark: significant progress towards correct solution
	(iii) PQ has $m=\frac{-1}{3}$ and passes through $(3,3)$ Using $y-3=\frac{-1}{3}(x-3)$, line is $3y-9=-x+3$ $\therefore PQ$ is the line $x+3y-12=0.$	1 mark : correct solution
	(iv) $dPQ=\sqrt{3^2+1^2}$ $=\sqrt{10}$	1 mark : correct answer
	(v) Using $d=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ with $x+3y-12=0$ and $A(-1,-3)$, $d=\frac{\lvert -1+3(-3)-12\rvert}{\sqrt{1^2+3^2}}=\frac{\lvert -22\rvert}{\sqrt{10}}=\frac{22}{\sqrt{10}}$ units	2 marks : correct solution 1 mark: significant progress towards correct solution
	(v) Area $\Delta APQ=\frac{1}{2}\times PQ\times d$ $=\frac{1}{2}\times\sqrt{10}\times\frac{22}{\sqrt{10}}$ = 11 units ^{2}	1 mark : correct answer

H5	(c) $x^2 + y^2 = 4$ is a circle of radius 2, centre $(0,0)$. $x + y = 2$ is the line through $(2,0)$ and $(0,2)$. $(0,0)$ satisfies both $x^2 + y^2 \le 4$ and $x + y \le 2$  	2 marks : correct solution 1 mark: significant progress towards correct solution
H5	(d) As the diagonals bisect, it is a parallelogram. A parallelogram with a right angle is a rectangle. $\therefore$ it is a rectangle **Comments:** • **AAA or SAS are not to be used as tests for similar triangles. The correct tests are "Equiangular" and "Two pairs of sides in the same ratio and included angles equal".** • **In part (d) when asked to comment on the significance of the information marked in the quadrilateral, many students only listed what was marked.** **You needed to comment on the significance of the diagonals bisecting (which was that this made it a parallelogram), and of one angle being a right angle.** **Very few students appeared to know the definition of a rectangle (a parallelogram containing a right angle).**	2 marks : correct solution 1 mark: significant progress towards correct solution or correct answer without correct justification

Question No. 12 Solutions and Marking Guidelines

Outcomes Addressed in this Question

H5 applies appropriate techniques from the study of calculus, to solve problems

H8 uses techniques of integration to calculate areas and volumes

Outcome	Solutions	Marking Guidelines
H8	**(a) (i)** $y = x\sin x$	**1 mark** Both correct values in exact form.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{\pi\sqrt{2}}{8}$	$\frac{\pi}{2}$	$\frac{3\pi\sqrt{2}}{8}$	0

H8 **(ii)**

$$\int_0^{\pi} x\sin x\,dx \approx \frac{\pi/2}{6}\left(0 + 4\times\frac{\pi\sqrt{2}}{8} + \frac{\pi}{2}\right) + \frac{\pi/2}{6}\left(\frac{\pi}{2} + 4\times\frac{3\pi\sqrt{2}}{8} + 0\right)$$

$$= \frac{\pi}{12}\left(\frac{\pi\sqrt{2}}{2} + \frac{\pi}{2}\right) + \frac{\pi}{12}\left(\frac{\pi}{2} + \frac{3\pi\sqrt{2}}{2}\right)$$

$$= \frac{\pi}{12}\left(\frac{4\pi\sqrt{2}}{2} + \frac{2\pi}{2}\right)$$

$$= \frac{\pi}{12}\left(2\pi\sqrt{2} + \pi\right)$$

$$= \frac{2\pi^2\sqrt{2}}{12} + \frac{\pi^2}{12}$$

$$= \frac{\pi^2\left(2\sqrt{2} + 1\right)}{12}$$

2 marks Correct solution. **1 mark** Substantial progress towards correct solution.

H8 **(b) (i)** Sphere

1 mark Correct answer

H8 **(ii)** Since the curve being rotated about the x-axis has line symmetry about the y axis, we can simply calculate the volume created by rotating half the curve about the x-axis and then double this answer to give the total volume created.

1 mark Correct answer, noting the symmetry of the curve being rotated.

H8 **(iii)**

$$V = 2\pi\int_0^r \left(r^2 - x^2\right)dx$$

$$= 2\pi\left[r^2x - \frac{x^3}{3}\right]_0^r$$

$$= 2\pi\left[r^3 - \frac{r^3}{3}\right]$$

$$= 2\pi\times\frac{2r^3}{3}$$

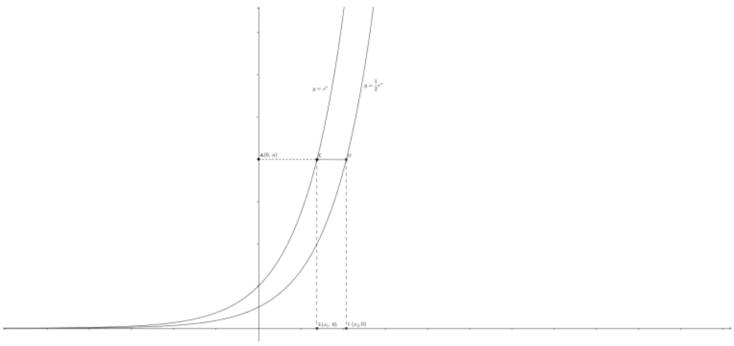
$$= \frac{4\pi r^3}{3}\text{units}^3 \quad \text{as required}$$

2 marks Correct solution. **1 mark** Substantial progress towards a correct solution

H5	**(c)** $f''(x)=6x$ $f'(x)=3x^2+c_1$ but $f'(0)=-2$ $\therefore -2=0+c_1$ $c_1=-2$ $\therefore f'(x)=3x^2-2$ $f(x)=x^3-2x+c_2$ but $f(1)=0$ $\therefore 0=1-2+c_2$ $c_2=1$ $\therefore f(x)=x^3-2x+1$	**2 marks** Substantially correct solution clearly showing the evaluation of the two constants. **1 mark** Substantial progress towards a correct solution.
H5	**(d)** $\int 4xe^{x^2}\,dx=2e^{x^2}+c$	**1 mark** Correct answer.
H8	**(e)** $y=4-x^2 \quad \Rightarrow \quad x^2=4-y$ $y=4-2x \quad \Rightarrow \quad 2x=4-y$ $x=\frac{4-y}{2}$ $x^2=\frac{(4-y)^2}{4}$ $V=\pi\int_0^4 4-y\,dy-\pi\int_0^4\frac{(4-y)^2}{4}dy$ $=\pi\left[4y-\frac{y^2}{2}\right]_0^4-\pi\left[\frac{(4-y)^3}{12}\right]_0^4$ $=\pi(8)-\pi\left(\frac{16}{3}\right)$ $=\frac{8\pi}{3}$ units^3	**3 marks** Correct solution **2 marks** Substantial progress towards correct solution, showing correct process including finding primitive functions. **1 mark** Some progress towards a correct solution showing the correct functions to be integrated
H8	**(f)** The area can be calculated as the area under the section of the parabola plus the area of two triangles. AC cuts the x-axis at 3 and C has co-ordinates (4, 4). $A=\left\lvert\int_0^2 -x^2\,dx\right\rvert+\frac{1}{2}\times1\times4+\frac{1}{2}\times1\times4$ $=\left\lvert\left[\frac{-x^3}{3}\right]_0^2\right\rvert+4$ $=\frac{8}{3}+4$ $=\frac{20}{3}$ units^2	**2 marks** Correct solution **1 mark** Substantial progress towards correct solution

Year 12 Question No. 13	Mathematics Solutions and Marking Guidelines	Trial HSC 2012
Outcomes Addressed in this Question		
H3 manipulates algebraic expressions involving logarithmic and exponential functions		

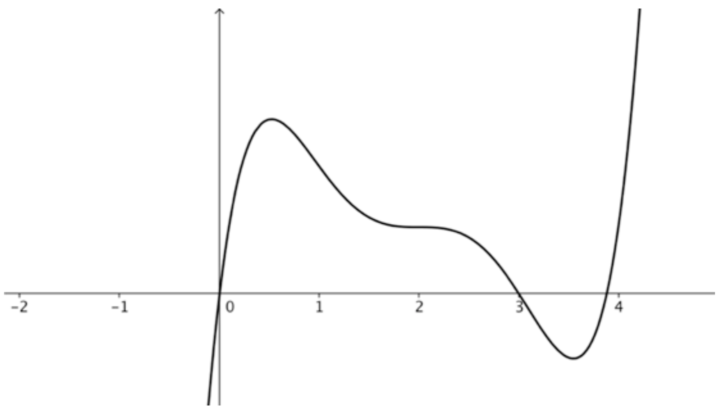
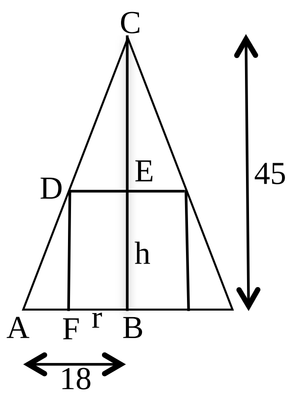
	Solutions	Marking Guidelines
(a)	$\log_a 2 = x$ and $\log_a 3 = y$ $\log_a 12 = \log_a (3 \times 4)$ $= \log_a (3) + \log_a (4)$ $= \log_a (3) + 2\log_a (2)$ $= y + 2.x$	**Award 2** Correct solution. **Award 1** Attempting to use an appropriate process.
(b)	$y = 3\ln x + 2$ $\frac{dy}{dx} = \frac{3}{x}$ At $(1,2)$, $\frac{dy}{dx} = \frac{3}{1} = 3 = m_{\text{tangent}}$ Equation of tangent $y - 2 = 3(x - 1)$ $\Rightarrow y = 3x - 1$ $\Rightarrow 3x - y - 1 = 0$	**Award 2** Correct solution. **Award 1** Attempts to find the equation of tangent.
(c)	$\text{LHS} = \frac{d}{dx}\left[\frac{e^{2x}}{2x+1}\right]$ $= \frac{(2x+1).2e^{2x} - e^{2x}.2}{(2x+1)^2}$ $= \frac{4xe^{2x} + 2e^{2x} - 2e^{2x}}{(2x+1)^2}$ $= \frac{4xe^{2x}}{(2x+1)^2}$ $= \text{RHS}$	**Award 2** Correct solution. **Award 1** Attempts to find the derivative of the given expression by an appropriate method.
(d)	$2e^{3x} - e^{2x} = 0$ $\therefore e^{2x}(2e^x - 1) = 0$ $\therefore e^{2x} = 0 \rightarrow$ no solution or $\therefore 2e^x - 1 = 0 \Rightarrow e^x = \frac{1}{2}$ $\therefore x = \ln\left(\frac{1}{2}\right) = -\ln 2$	**Award 2** Correct solution. **Award 1** Attempts to solve the given equation by an appropriate method.

(e) (i)	$\text{LHS} = \frac{d}{dx}\left[(x-1)\log_e 2x\right]$ $= (x-1).\frac{2}{2x} + \log_e 2x.1]$ $= \frac{x-1}{x} + \log_e 2x$ $= \text{RHS}$	**Award 1** Correct solution
(ii)	$\int_{\frac{1}{2}}^{1} \log_e 2x\, dx = \int_{\frac{1}{2}}^{1} \left\{\frac{d}{dx}\left[(x-1)\log_e 2x\right] - \frac{x-1}{x}\right\} dx$ $= \left[(x-1)\log_e 2x\right]_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} \frac{x-1}{x}\, dx$ $= 0 - \left(-\frac{1}{2}\log_e 1\right) - \int_{\frac{1}{2}}^{1} \left\{1 - \frac{1}{x}\right\} dx$ $= -\left[x - \ln x\right]_{\frac{1}{2}}^{1}$ $= -\left\{(1-\ln 1) - \left(\frac{1}{2} - \ln\frac{1}{2}\right)\right\}$ $= -\frac{1}{2} - \ln\frac{1}{2}$ $= \ln 2 - \frac{1}{2}$	**Award 3** Correct solution. **Award 2** Substantial progress towards solution. **Award 1** Limited progress towards solution.
(f) (i)		**Award 1** Graph represents situation as stated
(ii)		**Award 2** Correct solution. **Award 1** Substantial progress towards solution..

	Let $C(x_1, a)$ and $D(x_2, a)$ $\therefore C(x_1, e^{x_1})$ and $D(x_2, \frac{1}{2}e^{x_2})$ $CD = \lvert x_2 - x_1 \rvert$ $e^{x_1} = a \Rightarrow x_1 = \ln a$ $\frac{1}{2}e^{x_2} = a \Rightarrow x_2 = \ln 2a$ $\therefore CD = \lvert x_2 - x_1 \rvert = \lvert \ln 2a - \ln a \rvert$ $= \left\lvert \ln\left(\frac{2a}{a}\right) \right\rvert$ $= \lvert \ln 2 \rvert$ $\therefore CD = \ln 2$ ...... which is a constant.	

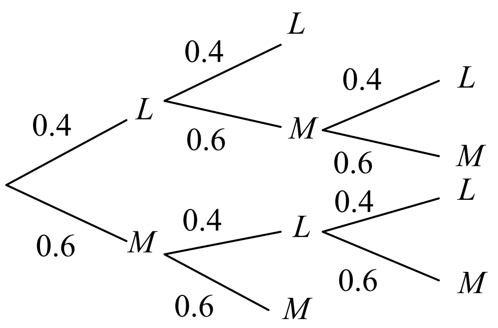
Year 12 Mathematics HSC Trial Examination 2012		
Question No. 14	Solutions and Marking Guidelines	
Outcomes Addressed in this Question		

H2 constructs arguments to prove and justify results
H5 applies appropriate techniques from the study of calculus and geometry to solve problems
H6 uses the derivative to determine the features of the graph of a function

Outcome	Solutions	Marking Guidelines
H2, H5	**(a) (i)** When $x = 2$, $g'(x) = 0$ ie. a stationary point exists at $x = 2$ To classify the stationary point, consider the following table, where value have been taken from the graph: x: 1.5, 2, 3 $g'(x)$: -ve, 0, -ve Slopes of gradients:  $\therefore$ There is a horizontal point of inflexion at $x = 2$.	**2 marks** Correctly identifies feature with valid justification.. **1 mark** Correctly identifies feature without justification OR only partially identifies feature but justification is valid
H6	**(ii)** A possible graph for $y = g(x)$ 	**2 marks** Graph drawn through given point, showing three main features in their correct place. **1 mark** Graph drawn is substantially correct.
H2	**(iii)** Being given the point $g(0) = 0$ provides a value for the constant when the primitive of the function $y = g'(x)$ is graphed. The graph then has a defined point of reference on the y-axis.	**1 mark** Given answer demonstrates a valid understanding.
H2	**(b) (i)** Δ's ABC and AFD are similar. As ratios of corresponding sides in similar triangles are equal: $\frac{DF}{CB} = \frac{AF}{AB}$	**2 marks** Correct justification noting both that triangles are similar and the relevant property. **1 mark** Identifies similar triangles but not relevant property.
H5	**(ii)** Taking a cross section of the cone through the vertex 	**2 marks** Correct solution. **1 mark** Substantial progress towards a correct solution

	Using similar triangles as in part (i) $\frac{DF}{CB}=\frac{AF}{AB}$ $\frac{h}{45}=\frac{18-r}{18}$ $h=\frac{45(18-r)}{18}$ $=\frac{5(18-r)}{2}$ as required.	
H5	**(iii)** Volume of cylinder $V=\pi r^2 h$ $=\pi r^2.\left(\frac{5(18-r)}{2}\right)$ $=\frac{5\pi r^2(18-r)}{2}$ $=\frac{90\pi r^2-5\pi r^3}{2}$ $\frac{dV}{dr}=\frac{180\pi r-15\pi r^2}{2}$ $\qquad$ $\frac{d^2V}{dr^2}=\frac{180\pi-30\pi r}{2}$ $=30\pi-5\pi r$ Max./Min. will occur when $\frac{dV}{dr}=0$ ie. $\frac{60\pi r-5\pi r^2}{2}=0$ $5\pi r(12-r)=0$ $r=0,12$ When $r = 0$ $\qquad$ $\frac{d^2V}{dr^2}=30\pi>0$ $\qquad$ $\therefore$ Minimum When $r = 12$ $\frac{d^2V}{dr^2}=30\pi-60\pi$ $=-30\pi<0$ $\qquad$ $\therefore$ Maximum $\therefore$ Cylinder will have a maximum volume when r = 12cm.	**3 marks** Correct solution giving correct answer and justification of answer as a maximum. **2 marks** Substantially correct solution and justification. **1 mark** Makes some valid progress towards a correct solution.
H2	**(c) (i)** $f(x)=3x^2-6x+7$ Positive definite if $a>0$ and $\Delta<0$ $a=3$ $\qquad$ $\therefore a>0$ $\Delta=b^2-4ac$ $=36-4\times3\times7$ $=-48$ $\qquad$ $\therefore \Delta<0$ $\therefore$ Function is positive definite for all real values of x.	**1 mark** Correctly shows function is positive definite.
H2, H5	**(ii)** $g(x)=x^3-3x^2+7x-10$ $g'(x)=3x^2-6x+7$ Now, $g'(x)>0$ for all real values of x ie. gradient of $g(x)>0$ for all real values of x $\therefore g(x)$ is increasing for all real values of x	**2 marks** Correct solution using calculus to justify that function is increasing, making link back to part (i) OR other correct and justifiable means. **1 mark** Substantial progress towards the required result.

Year 12 TRIAL Question No. 15	Mathematics Solutions and Marking Guidelines	Examination 2012
Outcomes Addressed in this Question		
H5 – applies appropriate techniques from the study of probability and series to solve problems.		
Outcome	**Solutions**	**Marking Guidelines**
H5	**a) (i)** $a=7,\ d=4$ $T_n = a+(n-1)d$ $111 = 7+(n-1)4$ $104 = 4n-4$ $108 = 4n$ $n = 27$ $\therefore$ yes 111 is the 27th term in this sequence.	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution
H5	**(ii)** $S_n = \frac{n}{2}(a+l) \qquad l = 111-4 = 107$ $S_{26} = \frac{26}{2}(7+107)$ $= 1482$	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution
H5	**b) (i)** The first \$2400 is invested for 3 years and amounts to: $A_1 = 2400(1+0.0725)^3$ The second \$2400 is invested for 2 years and amounts to: $A_2 = 2400(1+0.0725)^2$ The third \$2400 is invested for 1 year and amounts to: $A_3 = 2400(1+0.0725)^1$ $\therefore$ After three years: $A_1+A_2+A_3 = \$8\,295.37$	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution
H5	**(ii)** The first \$2400 is invested for 40 years and amounts to: $A_1 = 2400(1+0.0725)^{40}$ The second \$2400 is invested for 39 years and amounts to: $A_2 = 2400(1+0.0725)^{39}$ The third \$2400 is invested for 38 years and amounts to: $A_3 = 2400(1+0.0725)^{38}$ **...** The last \$2400 is invested for 1 year and amounts to: $A_{40} = 2400(1+0.0725)^1$ **After 40 years, James will have:** $2400(1+0.0725)^{40}+2400(1+0.0725)^{39}+2400(1+0.0725)^{38}+...+$ $2400(1+0.0725)^1$ $=2400(1.0725^1+1.0725^2+1.0725^3+...+1.0725^{40})$ ****This is a G.P with** $a=1.0725 \quad r=1.0725 \quad S_n=\frac{a(r^n-1)}{r-1}$ $=2400\left(\frac{1.0725(1.0725^{40}-1)}{1.0725-1}\right)$ $=\$548160.10$	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution

	$\therefore$ James will be able to retire after 40 years service as he will have more than \$500 000 in his superannuation fund.	
H5	**c)** **(i)** $r=\sqrt{11}-3$ ≈ 0.3166 Since $-1<r<1$ the geometric series has a limiting sum.	**(1 mark)** correct answer
H5	**(ii)** $S=\dfrac{a}{1-r}$ $=\dfrac{1}{1-\left(\sqrt{11}-3\right)}$ $=\dfrac{1}{4-\sqrt{11}}\times\dfrac{4+\sqrt{11}}{4+\sqrt{11}}$ $=\dfrac{4+\sqrt{11}}{16-11}$ $=\dfrac{4+\sqrt{11}}{5}$	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution
	d)  	
H5	**(i)** P(two sets only) = P(LL) + P(MM) $=(0.4)^2+(0.6)^2$ $=0.52$	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution
H5	**(ii)** P(Lisa wins the match) = P(LL) + P(LML) + P(MLL) $=(0.4)^2+(0.4\times0.6\times0.4)+(0.6\times0.4\times0.4)$ $=0.352$	**(2 marks)** correct solution with working. **(1 mark)** substantial progress towards correct solution

Year 12 Question No. 16	Mathematics Solutions and Marking Guidelines	Trial HSC 2012

Outcomes Addressed in this Question

P4	chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
H2	constructs arguments to prove and justify results
H5	applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems

	Solutions	Marking Guidelines
(a)	$2\sin\theta-\sqrt{3}=$ $\sin\theta=\dfrac{\sqrt{3}}{2}$ $\therefore\ \theta=60^\circ,120^\circ$	**Award 2** Correct solution. **Award 1** Attempting to find θ by an appropriate process.
(b)	Smallest angle, θ, is opposite shortest side $\therefore \cos\theta=\dfrac{34^2+27.4^2-24.3^2}{2\times34\times27.4}\approx0.7064566337$ $\therefore\ \theta\approx45.05265626=45^\circ3'$ (nearest minute)	**Award 2** Correct solution. **Award 1** Attempts to find an angle.
(c) (i)	$15=r\times\dfrac{\pi}{5}$ $\therefore\ r=\dfrac{75}{\pi}$ cm ≈ 23.87324146 cm	**Award 1** Correct solution.
(ii)	$A=\dfrac{1}{2}\times\left(\dfrac{75}{\pi}\right)^2\times\dfrac{9\pi}{5}\approx1611\cdot4\,\text{cm}^2$	**Award 2** Correct solution. **Award 1** Attempts to find the area by an appropriate method.
(d)	$\cos\dfrac{\pi}{4}+\sin\dfrac{2\pi}{3}=\dfrac{1}{\sqrt{2}}+\dfrac{\sqrt{3}}{2}$ $=\dfrac{\sqrt{2}}{2}+\dfrac{\sqrt{3}}{2}$ $=\dfrac{\sqrt{2}+\sqrt{3}}{2}$	**Award 2** Correct solution. **Award 1** Finds the exact value of only one angle.
(e) (i)	$\text{LHS}=\dfrac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}$ $=\dfrac{\sec^2\theta.\cot\theta}{\text{cosec}^2\theta}$ $=\dfrac{\sin^2\theta.\cot\theta}{\cos^2\theta}$ $=\tan^2\theta.\dfrac{1}{\tan\theta}$ $=\tan\theta$ $=\text{RHS}$	**Award 2** Correct solution. **Award 1** Substantial progress towards solution.

(f) (i)	$y = \frac{x^2}{8} - 1$ $\frac{x^2}{8} = y + 1$ $x^2 = 8(y+1)$ Compare with $(x-h)^2 = 4a(y-k)$ a parabola with vertex (h,k) and focal length $= a$ $\therefore$ Vertex $= (0, -1)$.	**Award 1** Correct solution
(ii)	From the form given in (i), $4a = 8 \Rightarrow a = 2$.	**Award 1** Correct solution
(g)	$4x^2 - mx + 9 = 0$ $\therefore \Delta = (-m)^2 - 4 \times 4 \times 9 = m^2 - 144$ To have exactly one real root, Δ=0. $\therefore m^2 - 144 = 0$ $\therefore m = \pm 12$	**Award 2** Correct solution. **Award 1** Attempting to find *m* by an appropriate process.